Quiz 4.2: Sample Answers

1. Find the derivative of $y = \cos(e^{9x})$.

   Using chain rule, we get:

   $$\begin{aligned} y' &= -\sin(e^{9x})(e^{9x})' \\ &= -\sin(e^{9x})(9e^{9x}) \end{aligned}$$

2. Find the derivative of $y = (e^{-6x})(\sin 6x)$.

   Using product rule, we get:

   $$y' = (e^{-6x})(\sin 6x)' + (e^{-6x})'(\sin 6x)$$

   Then using chain rule for each derivative, we get

   $$y' = (e^{-6x})(6\cos 6x) + (-6e^{-6x})(\sin 6x)$$

3. Find the derivative of $y = (x^6 + x)^4$.

   Using the power rule, we get:

   $$\begin{aligned} y' &= 4(x^6 + x)^3(x^6 + x)' \\ &= 4(x^6 + x)^3(6x^5 + 1) \end{aligned}$$

4. Find the equation of the tangent line to $y = e^x \cos 6x - 2$ at the point $(0, -1)$.

   We first find the derivative using product rule then chain rule:

   $$\begin{aligned} y' &= (e^x)(\cos 6x)' + (e^x)'(\cos 6x) \\ &= (e^x)(-6\sin 6x) + (e^x)(\cos 6x) \end{aligned}$$

   So, substituting $x = 0$, we get the slope of the tangent line as

   $$m = (e^0)(-6\sin 6(0)) + (e^0)(\cos 6(0)) = (1)(0) + (1)(1) = 1$$

   We then substitute $x = 0$, $y = -1$, and $m = 1$ into $y = mx + b$ to get $b = -1$. So the equation of the tangent line is $y = x - 1$.

5. Find the velocity at time t of a vibrating string whose displacement is given by the equation $s = 9 + \frac{1}{3}\sin(9\pi t)$.

   We simply have to find the derivative:

$$s'(t) = \frac{1}{3}(\cos 9\pi t)(9\pi t)' = \frac{1}{3}(\cos 9\pi t)(9\pi) = 3\pi \cos 9\pi t$$

   So the velocity at time t is $3\pi \cos 9\pi t$.

6. Find the derivative of $f(x) = \sin(2\cos x)$.

   We use chain rule:

$$f'(x) = \cos(2\cos x)(2\cos x)' = \cos(2\cos x)(-2\sin x)$$